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An Optimal Strategy to Change the Non-Changeable Assets

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Abstract

This paper introduces a novel strategy for modifying two ostensibly immutable assets in the market, employing the exchange option modeled and solved through a mesh-free numerical method based on radial basis functions. A specific currency market scenario is designed to assess its performance where two currencies remain unalterable. The strategy is then applied to this controlled environment, resulting in a redefined structure for exchange currency pricing. The numerical results, presented using MATLAB, illustrate the practicality and efficacy of our approach, contributing both a strategic framework for altering traditionally non-changeable assets and showcasing rigorous modeling and numerical methods in an academic context.

Keywords: Exchange option, Currency market, Mesh-free numerical method, Asset modification, Financial modeling, Controlled scenario.

1 | Introduction

Exchange option is a major instrument in financial markets. As it is implied by the word "Option", exchange option is a conditional claim whose owner has the option to change his/her own asset into another asset at the due date or not. However, sometimes the situation of the market proceeds in a way that it is not possible to exchange two assets. So, there is no way to define financial instruments like exchange option for them. For a better understanding, suppose that country X refuses to accept country Y's currency due to various political and economic reasons. Therefore, country Y could not have direct access to country X's currency and/or

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establish agreements that exchange these two currencies directly, so exchange option wouldn't be established on these two currencies. The aim of this paper is to present a solution for such this situation. For this purpose, we study the currencies under exchange option feature. in fact, we price exchange option using financial mathematics concepts, present a solution for that, and finally apply the currencies model to the exchange option. In fact, another approach for currency options price is presented in this study based on previous literature.

The exchange option is a highly interesting instrument presented and formulated by Margrabe [1]. Various studies were conducted on modeling these securities, including Geman et al. [2], Antonelli et al. [3], Cheang and Chiarella [4], and Wang [5]. German et al. [2] created a closed formula for exchange options using "change of numéraire." Also, Kim and Koo [6] created a close formula for the situation that drifts are two same assets using Mellin transform. Antonelli et al. [3] considered another volatility structure and priced these kinds of options. Cheang and Chiarella and also Wang modeled power exchange options considering Jump. Two of the most significant studies on currency option pricing are Biger and Hull [7] and Geman and Kohlhagen [8] who managed to model and formulate it. other studies include Nafei et al. [9], Baharumshah et al. [10], Azizi et al. [11] and Yu-Min Lian et al. [12].

As mentioned, we are searching for a solution to define exchange option on two non-changeable assets. As facing such a situation in the real world is inevitable, presenting a solution for this problem is essential. These problems are tangible in world markets, which urges more studies and research in the field. Therefore, we approach mathematics modeling to address such problems.

To this end, in the next chapter, we present the exchange option structure, modeling, and pricing of these securities. The third chapter provides a solution for designing an exchange option on two non-changeable assets. The fourth chapter examines and applies these situations to the special status of currency markets.

2 | Exchange Option

In this part, we model and price the exchange option. To this end, two random assets are chosen, and then the Exchange Option is used as the U function for exchanging these two assets. After modeling, we provide a solution and perform the model using MATLAB software.

Suppose that $(\Omega, \Im, Q)$ is the probability space, which the model is performed and consider the price dynamics of two random assets:

$$\begin{aligned} dS(t) &= \alpha_1 S(t)dt + \sigma_1 S(t)dw_t^1, \\ dS^*(t) &= \alpha_2 S^*(t)dt + \sigma_2 S^*(t)dw_t^2. \end{aligned} \quad (1)$$

in which $\alpha_i, i \in \{1,2\}$ is drift, $\sigma_i, i \in \{1,2\}$ is volatility, and w_t^1 and w_t^2 are the wiener processes, so that:

$$dw_t^1 dw_t^2 = \rho dt.$$

Assume that $U(S, S^*, t)$ is the exchange option price of two assets with S and S^* prices at time t. Its differential would be as follows via Itô calculus [13]:

$$dU = \begin{pmatrix} U_t + \alpha_1 S U_S \\ +\alpha_2 S^* U_{S^*} + \sigma_1^2 S^2 U_{SS} \\ +\sigma_2^2 S^{*2} U_{S^*S^*} + \rho\sigma_1\sigma_2 S S^* U_{SS^*} \end{pmatrix} dt + \left(\sigma_1 S U_S + \sigma_2 S^* U_{S^*}\right) dW. \quad (2)$$

By using the Feynman–Kac formula, we could reach the following PDE [13]:

$$\frac{\partial U}{\partial t}+\alpha_1 S\frac{\partial U}{\partial S}+\alpha_2 S^*\frac{\partial U}{\partial S^*}+\frac{1}{2}\sigma_1^2 S^2\frac{\partial^2 U}{\partial S^2}+\rho\sigma_1\sigma_2 SS^*\frac{\partial^2 U}{\partial S\partial S^*}+\frac{1}{2}\sigma_2^2 S^{*2}\frac{\partial^2 U}{\partial S^{*2}}-rU=0. \quad (3)$$

that its terminal condition would be as follows:

$$U\left(S,S^*,T\right)=(S^*-S)^+. \quad (4)$$

2.1 | PDE Solution

In this part, we intend to solve exchange option PDE (3) using numerical methods and use heat equation solution using RBF for solving PDE. We acquire exchange option prices using options pricing methods in De Marchi et al. [14] studies. To this end, the change of the variables as $\tau = T - t$ and $C(S,S^*,\tau) = U(S,S^*,t)$ is considered, and the PDE is rewritten as follows:

$$C_\tau = f_1\left(S\right)C_S + f_2\left(S^*\right)C_{S^*} + f_3\left(S\right)C_{SS} + f_4\left(S\right)C_{S^*S^*} + f_5\left(S,S^*\right)C_{SS^*} - rC. \quad (5)$$

in which:

$$f_1(S)=\mu_1 S, f_2(S^*)=\mu_2 S^*, f_3(S^*)=\frac{1}{2}\sigma_1^2 S^2,$$
$$f_4(S^*)=\frac{1}{2}\sigma_2^2 S^{*2}, f_5(S,S^*)=\rho\sigma_1\sigma_2 SS^*. \quad (6)$$

Suppose $S_1 \le \cdots \le S_i \le \cdots \le S_m$ and $S_1^* \le \cdots \le S_j^* \le \cdots \le S_n^*$ are the discrete points, so that we have points as $W_l = \left(S_i, S_j^*\right)$ and ϕ is a Multiquadric (MQ) function as follows:

$$\phi(r)=\sqrt{1+(\varepsilon r)^2}. \quad (7)$$

in which:

$$r=\|W\|. \quad (8)$$

Then, assuming $K = mn$ and $C=\sum_{l=1}^{K}\omega_l\left(\tau\right)\phi\left(\|W-W_l\|\right)$, the PDE derivatives would be as follows:

$$C_\tau=\sum_{l=1}^{K}\dot{\omega}_l\left(\tau\right)\phi\left(\|W-W_l\|\right), C_S=\sum_{l=1}^{K}\omega_l\left(\tau\right)\phi_S\left(\|W-W_l\|\right),$$
$$C_{S^*}=\sum_{l=1}^{K}\omega_l\left(\tau\right)\phi_{S^*}\left(\|W-W_l\|\right), C_{SS}=\sum_{l=1}^{K}\omega_l\left(\tau\right)\phi_{SS}\left(\|W-W_l\|\right), \quad (9)$$
$$C_{S^*S^*}=\sum_{l=1}^{K}\omega_l\left(\tau\right)\phi_{S^*S^*}\left(\|W-W_l\|\right), C_{SS^*}=\sum_{l=1}^{K}\omega_l\left(\tau\right)\phi_{SS^*}\left(\|W-W_l\|\right).$$

in which $\| \cdot \|$ is an euclidean norm, $(W-W_l)=(S-S_i, S^*-S_j^*)$ and the feature of derivatives would be as follows:

Other derivatives could be achieved exactly as above. It is possible to make $L, L_S, L_{S^*}, L_{SS}, L_{S^*S^*}, L_{SS^*}$ matrices using all the points so that the elements of the matrix would be as follows:

and consider ω as follows:

$$\omega=[\omega_1,\cdots,\omega_K]^{\text{Transpose}}. \quad (12)$$

With scientific notation, we have the following ODE:

$$\begin{aligned}
\phi_S(\|(W-W_l)\|) &= \frac{-\varepsilon(S+S_i)}{(\varepsilon(S-S_i)^2+\varepsilon(S^*-S_j^*)^2+1)^{\frac{3}{2}}},\\
\phi_{SS}(\|(W-W_l)\|) &= \frac{-\varepsilon(\varepsilon S^{*2}+2\varepsilon S^* S_j^* - 2\varepsilon S^2 - 4\varepsilon S S_i + \varepsilon S_j^{*2} - 2\varepsilon S_i^2 + 1)}{(\varepsilon(S-S_i)^2+\varepsilon(S^*-S_j^*)^2+1)^{\frac{5}{2}}},\\
\phi_{SS^*}(\|(W-W_l)\|) &= \frac{3\varepsilon^2(S+S_i)(S^*+S_j^*)}{(\varepsilon(S-S_i)^2+\varepsilon(S^*-S_j^*)^2+1)^{\frac{5}{2}}}.
\end{aligned} \quad (10)$$

$$\begin{aligned}
&(L)_{ij}=\phi(\|W_i-W_j\|), (L_{SS})_{ij}=\phi_{SS}(\|W_i-W_j\|),\\
&(L_S)_{ij}=\phi_S(\|W_i-W_j\|), (L_{S^*S^*})_{ij}=\phi_{S^*S^*}(\|W_i-W_j\|),\\
&(L_{S^*})_{ij}=\phi_{S^*}(\|W_i-W_j\|), (L_{SS^*})_{ij}=\phi_{SS^*}(\|W_i-W_j\|).
\end{aligned} \quad (11)$$

$$\begin{aligned}
&L\dot{\omega}=\theta\omega,\\
&\omega(0)\leftrightarrow IN(C(S,S^*,0)).
\end{aligned} \quad (13)$$

in which IN in the initial condition function. Now using the Fasshauer et al. [15] study, we have the following algorithm:

2.2 | Algorithm

I. Consider Λ vector, including initial points corresponding $IN(C(S,S^*,0))$.

II. Acquire ω_τ from $\Lambda=L\omega_\tau$.

III. Acquire $\omega_{\tau+1}$ from $L\omega_{\tau+1}=[L+dt\theta]\omega_\tau$.

IV. Acquire $\Lambda=L\omega_{\tau+1}$ using $\omega_{\tau+1}$ and L (we don't consider Λ border points in the calculation).

V. Add border points to Λ.

VI. If $\tau<T$, then go to *Step 2.*

For a better understanding, we perform an example numerically. Assume that we have two asset prices in the interval of $[0,1]$, $m=n=21, \mu_1=\mu_2=0.1, \sigma_1=\sigma_2=0.2, r=0.1$, and perform the model using MATLAB software.

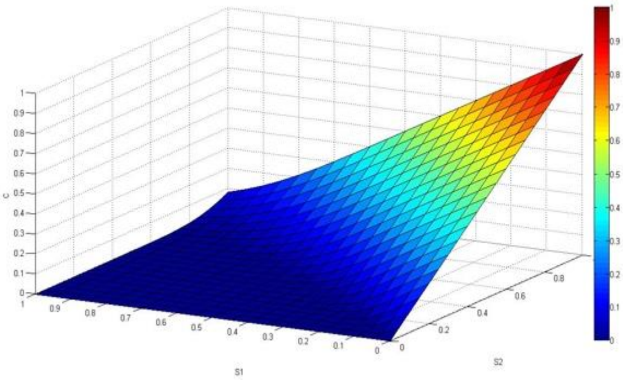

Fig. 1. Exchange option.

3 | Exchange Option on Two Non-Changeable Assets

Based on the previous parts, we will model special conditions of the exchange option. Sometimes, in markets, two assets are non-changeable; in other words, it is impossible to change one asset into another. For example, buying Bitcoin with Iran's Rial is impossible, so we use an intermediary currency like the U.S.

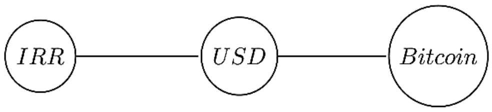

Fig 2. IRR to bitcoin.

Since these two assets are non-changeable, it is impossible to directly apply financial derivatives to them. In this part, we study and provide solutions for these non-changeable assets.

If S and S^* are non-changeable, we cannot define the exchange option. Suppose that $\bar{S}$ is an intermediary asset which is changeable with both S and S^* assets. To acquire the changing ability as an option, we need to Exchange options $C(S,\bar{S},t)$ and $C^*(\bar{S},S^*,t)$. Therefore, we form the $P(C,C^*,t)$ portfolio, which includes C and, C^* and regarding the structure of the portfolio, the payoff function is as follows:

$$\begin{cases} 0, & C(S(T),\bar{S}(T),T)=0 \vee C^*(\bar{S}(T),S^*(T),T)=0, \\ C(S(T),\bar{S}(T),T)+C^*(\bar{S}(T),S^*(T),T), & \text{otherwise.} \end{cases} \quad (14)$$

If we look deep, we find:

$$P(C,C^*,t)=\prod(S,\bar{S},S^*,t). \quad (15)$$

and the payoff would be as follows:

$$I_{S\le\bar{S}\le S^*}(S^*(T)-S(T)). \quad (16)$$

in which "I" is the indicator function, and we have the following regarding the measure Q feature:

$$\prod(S,\bar{S},S^*,t)=E^Q\begin{bmatrix}\prod(S(T),\bar{S}(T),S^*(T),T)\,| \\ S(t)=S_0,\bar{S}(t)=\bar{S}_0,S^*(t)=S_0^*\end{bmatrix}. \quad (17)$$

To better understand the portfolio, its figure is presented using MATLAB software and assuming $\mu_1 = \mu_2 = \mu_3 = 0.1, \sigma_1 = \sigma_2 = \sigma_3 = 0.2,$ assets price changes in interval $[0,1]$ and $dt = 0.01, dS = d\bar{S} = dS^* = 0.1.$ Using the Monte Carlo method, we can conclude as follows for pricing this portfolio:

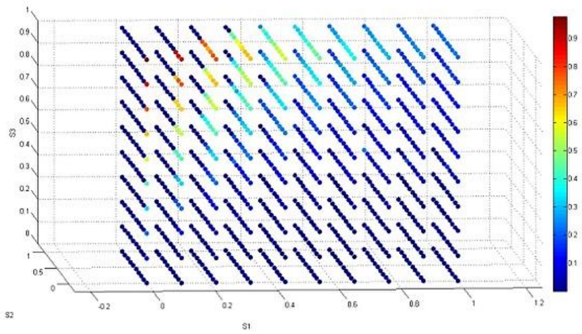

Fig. 3. Three assets exchange option.

The axis is an asset's price, and the point's color is the portfolio's value at the intersection of asset prices.

3.1 | Exchange Option on Two Non-Changeable Assets

Often, there are more than one intermediary asset between two considered assets in the market. Sometimes to change an asset to the other, we must add some more exchange options to the portfolio. For example, there are the following cases.

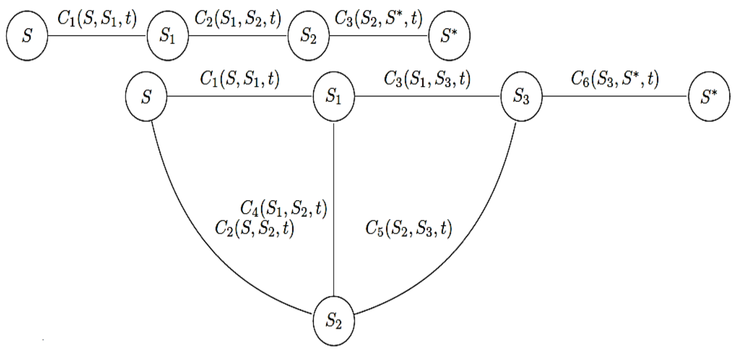

Fig. 4. More than one intermediary assets.

According to the above graph, it could be a couple (or more) of different paths for changing S to S^*. Each one of these paths has its portfolio. Assuming the $C_1,...,C_N$ options, the payoff of each portfolio is as follows:

$$\begin{cases} 0, & C_1(T) = 0, \text{or}, \cdots, \text{or}, C_N(T) = 0, \\ \sum_{i=1}^{N} C_i(T), & \text{otherwise}. \end{cases} \quad (18)$$

Each of these portfolios has its specific features. For example, if a portfolio has price exchange options , it means that the probability of changing the S asset to S^*asset gets higher because the high price exchange option shows the price difference between two assets in a way that the S^*asset is a far more valuable asset and the possibility of applying all the exchange options in this portfolio is higher than other portfolios in the due time.

Picking up a proper portfolio is essential, for the buyer's expectations might not be met, leading him/her toward loss. Each portfolio has a probability of applying and not applying. Each of these probabilities is closely associated with the option prices, so an optimized portfolio is essential. Buyer's expectations refer to his/her willingness to exchange the two assets. In other words, the price of the options inside the portfolio should be high. Different paths could be defined on this basis for different utilities. In the next part, we provide a proper method for choosing exchange options and discuss some points in choosing the options.

3.2| Finding an Optimized Path

In this part, we intend to choose one optimized path between a couple of underlying assets according to the buyer's utility, and to this end, consider the equivalent graph of each market, then via the buyer's utility in percent, could find the proper path.

For a better understanding, suppose that $S_1,...,S_N$ are intermediary assets considered the graph's nods. Now, we show the exchange options between two assets in the market with an edge between two nods.

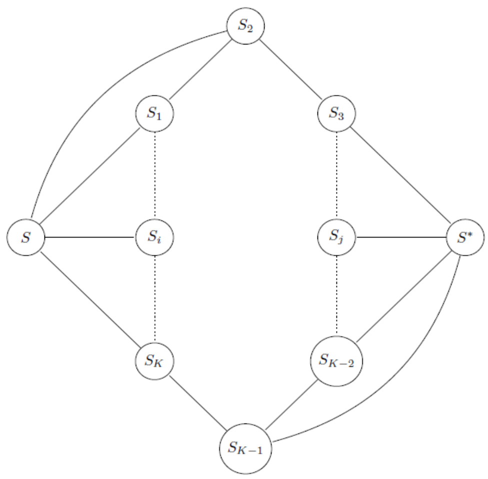

Fig. 5. Different paths between assets.

The maximum edges of the above model are $\frac{n(n+3)}{2}$. In fact, the maximum number of exchange options are $\frac{n(n+3)}{2}$. Suppose that the above graph has N edges and $C_1,...,C_N$ shows the exchange option prices or be the weight of the edges. Also, suppose that the buyer is 100 percent willing to exchange S to S^* asset, then we face a problem like this:

I. the minimum number of edges between S and S^*.

II. the heaviest edges.

To solve the above problem, the existence of the heaviest edges makes it difficult to find the most proper paths. To solve this, we changed the weight of the edges like this:

$$W_i := |C_i - \max\{C_1,...,C_N\}|,$$
$$i \in \{1,...,N\}. \quad (19)$$

The problem has turned into the shortest path problem, so we solve it with Dijkstra's algorithm and find the optimized path. Also, if the utility of this exchange is a% for the buyer, we could acquire the equivalent percentile of a%, and, again, weigh the edges with the reference method and find the optimized path.

3.3 | Algorithm

I. Consider the graph model market associated with the exchange option.

II. Price each exchange option related to two assets.

III. Consider a (the buyer's utility) and acquire the ath percent of the edges.
$q = \text{Percentile}(\{\text{set of wights}\}, \alpha)$

IV. Reweight the edges $W_j := |C_j - q|$.

V. Finding the shortest path with Dijkstra Algorithm.

4 | Currency Exchange

In this part, we will use the market data to price the currency options in another way. Using the amount of considered currency prices to be exchanged in a currency option and an index price, which considers the moment value. In this way, we consider a base currency for the beginning and compare other currencies' values according to the base, so we choose the US dollar. Suppose an investor provides options to exchange the Iranian Rial for Japanese Yen, and there is no exchange option for this. For the rest of the job, consider the hypothetical market model related to these currencies as follows:

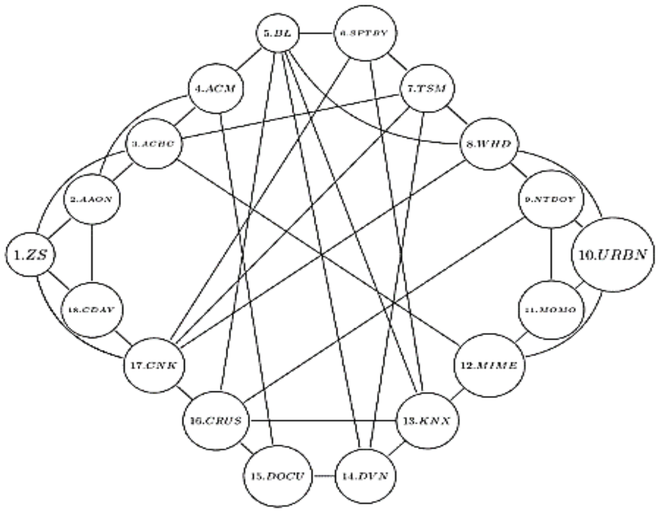

Fig. 6. Three assets exchange option.

Each of the edges of the above graph show two exchange option between the two currencies. For example, in the edge between IRR and CAD, there are two Exchange options from IRR to CAD and from CAD to IRR. The node name of each edge shows the national money of a country.

Table 1. National currency index.

Index	National Currency	Index	National Currency
IRR	Iran	JPY	Japan
CAD	Canada	GBP	England
KWD	Kuwait	DKK	Denmark
RUB	Russia	NZD	New Zealand
SGD	Singapore	KRW	South Korea
NOK	Norway	AUD	Australia
SEK	Sweden	OMR	Oman
CNY	China	TRY	Turkey
CHF	Swiss	EUR	Europe

Considering the price of the above currencies, we suppose its differential model follow geometric brownian motion and for estimating this model parameters, we use historical data that mentioned in the Bjork book (Arbitrage Theory in continuous time) [13], [16], [17]. Therefore, we extract the prices report from www.oanda.com in time interval of 1/1/2011 to 12/30/2018.

Table 2. Geometric brownian motion parameters.

Index	Drift	Volatility	Index	Drift	Volatility
IRR	0.0784	0.3572	JPY	0.0348	0.1636
CAD	0.0350	0.2239	GBP	0.0772	0.3824
KWD	0.0351	0.2390	DKK	0.0299	0.1796
RUB	0.0029	0.1170	NZD	0.0021	0.1637
SGD	0.0264	0.2321	KRW	0.0021	0.1910
NOK	0.0090	0.1729	AUD	0.0377	0.2836
SEK	0.0014	0.1508	OMR	0.0096	0.1886
CNY	0.0700	0.2375	TRY	0.0186	0.1284
CHF	0.0082	0.1791	EUR	0.0553	0.2132

Suppose that the established or contract price for exchanging the currencies per a unit of base currency USD in currency options in the market is as follows:

Table 3. Contract prices.

Currency	Price	Currency	Price
CAD	1.2701	NZD	1.4354
CHF	0.9867	NOK	8.1609
EUR	0.8492	OMR	0.385
GBP	0.7533	RUB	58.4102
JPY	112.8815	SGD	1.3543
AUD	1.3007	KRW	1.11e+03
CNY	6.6149	TRY	3.7996
DKK	6.3203	IRR	3.43e+04
KWD	0.3022	SEK	8.3141

These numbers are the prices that the option owner could pay to exchange the two assets regarding the graph at the time of the contract.

We use one unit of USD to evaluate the value changes of a currency. For example, suppose that the specified price is 1 unit, meaning that we can buy 1 unit of USD with 1 unit of the currency, and gradually, the price of each unit of USD changes to 1.1 (0.9) unit. Therefore, we need more currency for buying 1 unit of USD, and the value is decreased (increased) compared to the dollar. Therefore, regarding the established price, the value of the currency option changes. Given the effects of this approach received from the currency option contract and the live price in the market, applying this method for pricing the exchange option on currencies is more suitable, and the price realizes more. The advantage of this method over German and Kohlhagen [8] methods is that the buyer prices again using the existent indexes in the market and the information, then the option price gets closer to the real value.Using currency value, we price options or weight the edges using part two. The weight matrix related to the graph is shown using the following chart with color:

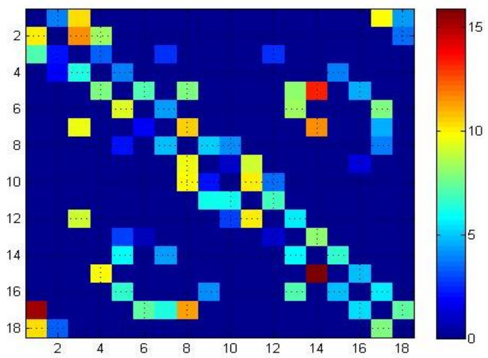

Fig. 7. Weight matrix.

The intersection of the two numbers (each number corresponding to each numbered) in the figure above shows the edge between the two nodes, the color of which, according to the side color bar, means the option price between them. Applying Dijkstra's algorithm and MATLAB software, we show the following results in terms of buyer utility:

Table 4. Contract prices.

Utility	Path
$\alpha \to 0$	S1 S3 S12 S10
$\alpha = 25\%$	S1 S2 S4 S5 S16 S9 S10
$\alpha = 50\%$	S1 S18 S17 S16 S5 S8 S10
$\alpha = 75\%$	S1 S17 S8 S10
$\alpha \to 100\%$	S1 S17 S8 S10

5 | Conclusions

This section delineates the process of introducing and modeling the exchange option within the context of two non-exchangeable assets. The problem formulation is intricately designed to encapsulate the essence of the exchange option definition, and a solution is derived through the acquisition of two Exchange options within a specified framework. The subsequent portfolio construction, incorporating two Exchange options , is meticulously addressed using Monte Carlo methods, and the resultant pricing is elucidated. The methodology is then extended to various forms, culminating in the application of real currency data to formulate portfolios transforming Iranian Rial to Japanese Yen, considering different utilities. The incorporation of the Dijkstra algorithm underscores the complexity and depth of our approach. A notable contribution of this study lies in the departure from conventional methods, particularly German and Kohlhagen's, for pricing currency options. This departure serves as a methodological innovation, offering a distinct perspective on currency option valuation. Notably, the study highlights the potential for further enrichment by integrating trade costs and components such as credit risk into the model, thereby elevating the research to a higher echelon of complexity and realism.

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Author Contributaion

Methodology, M. Nezhadkian.; Software, M. Nezhadkian. and M. Quan; Formal analysis, M. Quan.; Resources, M. Nezhadkian.and S.M. Azimi.; Data curation, S.M. Azimi. All authors have read and agreed to the published version of the manuscript.

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Data Availability

Not applicable.

Conflicts of Interest

The authors confirm the absence of any real or perceived conflicts of interest tied to the current study.

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