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Efficient Resource Allocation Management in Multicommodity Networks: A Fuzzy Multiobjective Approach

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Abstract

In this work, we consider a Multiobjective Minimal Cost Flow (MMCF) problem where there are several commodities to transport from sources to destinations and more than one conveyance for those transporting. We also assume that each commodity has distinct capacities in each conveyance. The obtained model is not necessarily balanced, and we introduced a method to solve this model without converting it to a balanced model. The advantages of the proposed method are also discussed.

Keywords: Fuzzy multiobjective solid minimal cost flow problem, LR flat fuzzy number, Multicommodity minimal cost flow problem.

1 | Introduction

Minimum Cost Flow (MCF) problems have many applications in almost all industries, such as agriculture, communications, education, energy, manufacturing, medicine, and transportation [1]. Generally, the MCF problem minimizes the cost of transporting products that are available at some sources and required at some destinations. However, there are few MCF problems with only a single objective in the real world. Therefore, in recent years, many authors have considered multiple objective minimum cost flow problems [2]. Another complexity in the real issues is the impreciseness of values of coefficients of the variables in the objective functions, availability, and demand of the products. The fuzzy set theory introduced by Zadeh [3] is a good

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alternative for this impreciseness. To our knowledge, Zimmermann [4] proposed the first formulation of fuzzy multiobjective linear programming. Also, for the first time, Shih and Lee [5] considered a fuzzy MCF problem. After that, this problem has been studied by many researchers from several viewpoints; see [6]–[8] and the references therein.

Our motivation for this paper is the recent works of Kaur and Kumar[8], [9]. In [9], the authors consider fuzzy multiobjective transportation problems where there exist some nodes, called intermediate nodes, at which the product may be stored in case of the excess of the available product, and later on, the product may be supplied from these intermediate nodes to the destinations. They assumed that there are different types of conveyances, such as trucks, cargo flights, trains, ships, etc., for transporting the products from sources to destinations. Such multiobjective transportation problems in which the conveyances and intermediate nodes are used simultaneously are known as multiobjective Solid Minimal Cost Flow (SMCF) problems [9]. The SMCF problem with fuzzy data was studied by several authors [9]–[13]. Sometimes, we must transport more than one commodity from sources to destinations. These problems are called multicommodity flow problems. Ghatee and Hashemi [14] studied fuzzy multicommodity flow problem, and Chakraborty et al. [15], Dalman et al. [16], Kundu et al. [17], and Rani et al. [18] considered multiobjective multi-item solid transportation problem under uncertainty. This paper considers a Fuzzy Multiobjective Multicommodity Minimal Cost Flow (FMMMCF) problem when conveyance limitations exist. In our model, a conveyance may be allowed to transport a certain amount of a commodity. To our knowledge, this model has no research, even for deterministic data.

This paper is organized into 6 sections. In the next section, some preliminaries of fuzzy numbers are reviewed. In Section 3, we describe our model, and a formulation of the FMMMCF problem is introduced. In Section 4, the new method is proposed, and we illustrate this method with some numerical examples in Section 5. The conclusion and some suggestions are given in Section 6.

2 | Preliminaries

In this section, we provide some preliminaries.

Definition 1 ([19]). A function L: $[0,\infty) \to [0,1]$ (or R: $[0,\infty) \to [0,1]$) is said to be a reference function of fuzzy numbers if and only if

I. $L(0) = 1$ (or $R(0) = 1$).

II. L (R) is non-increasing on $[0,\infty)$.

Definition 2 ([20]). A fuzzy number $\tilde{a} = (m, n, \alpha, \beta)_{LR}$ is said to be an LR flat fuzzy number if its membership function $\mu_{\tilde{a}}(x)$ is given by

$$\mu_{\tilde{a}} = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & \text{for } x \le m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & \text{for } x \ge n, \beta > 0 \\ 1, & \text{otherwise} \end{cases}$$

Definition 3 ([20]). Two LR flat fuzzy numbers $\tilde{a}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{a}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are said to be equal, i.e., $\tilde{a}_1 = \tilde{a}_2$ if and only if $m_1 = m_2$, $n_1 = n_2$, $\alpha_1 = \alpha_2$, and $\beta_1 = \beta_2$.

Definition 4 ([21]). An LR flat fuzzy number $\tilde{a} = (m, n, \alpha, \beta)_{LR}$ is said to be a non-negative LR flat fuzzy number if and only if $m - \alpha \ge 0$.

Let $\tilde{a}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{a}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two LR flat fuzzy numbers. Then

I. $\tilde{a}_1 \oplus \tilde{a}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$.

II. Let $\tilde{a}_1$ and $\tilde{a}_2$ be non-negative LR flat fuzzy numbers. Then

$$\tilde{a}_1 \otimes \tilde{a}_2 \simeq (m_1 m_2, n_1 n_2, (m_1 - \alpha_1)(m_2 - \alpha_2) - m_1 m_2, (n_1 + \beta_1)(n_2 + \beta_2) - n_1 n_2)_{LR}.$$

III. Let λ be a real number. Then

$$\lambda\tilde{a}_1 = \begin{cases} (\lambda m_1, \lambda n_1, \lambda \alpha_1, \lambda \beta_1)_{LR} & \lambda \geq 0, \\ (\lambda n_1, \lambda m_1, -\lambda \beta_1, -\lambda \alpha_1)_{LR} & \lambda < 0. \end{cases}$$
$$\lambda\tilde{a}_1 = \begin{cases} (\lambda m_1, \lambda n_1, \lambda \alpha_1, \lambda \beta_1)_{LR} & \lambda \geq 0, \\ (\lambda n_1, \lambda m_1, -\lambda \beta_1, -\lambda \alpha_1)_{LR} & \lambda < 0\,. \end{cases}$$

In this paper, we use modified Liou and Wang's [22] ranking for the comparison of fuzzy numbers.

Assume, $\tilde{a} = (m, n, \alpha, \beta)_{LR}$.

$$\Re(\tilde{a}) = \gamma \left[\int_0^1 (m\lambda + n(1-\lambda)) d\lambda \right] + (1-\gamma) \left[\lambda \int_0^1 \left(m - \alpha L^{-1}(\rho)\right) d\rho + (1-\lambda) \int_0^1 \left(n + \beta R^{-1}(\rho)\right) d\rho \right],$$

where $\gamma \in [0,1]$ and $\lambda \in [0,1]$.

Let $\tilde{a}$ and $\tilde{b}$ be two LR flat fuzzy numbers. Then $\tilde{a} \gtrsim \tilde{b}$ ($\tilde{a} \lesssim \tilde{b}$) if $\Re(\tilde{a}) \geq \Re(\tilde{b})$ $\left(\Re(\tilde{a}) \leq \Re(\tilde{b})\right)$.

3 | Fully Fuzzy Multicommodity Multiobjective Model

In this section, we introduce a fully fuzzy multicommodity multiobjective model for solid minimal cost flow problems in which there are limitations on conveyances for the transport of the products. For example, assume we want to transport coal and petroleum by train from one city to another. We need a special tank for each commodity (coal and petroleum). Therefore, we cannot allocate all the train's capacity to a commodity. For another example, assume that we want to send grease and petroleum from one country to another country by ship. Assume that some rules for importing grease or petroleum by ship in the destination country do not allow you to send more than a particular value of these materials. Therefore, you cannot allocate all the ship's capacity to a commodity. These examples show we must design a new model to cover these problems.

Assume that $G = (N, A)$ is a given network where N is the set of nodes and A is the set of links. We describe our problem with a simple example. Consider a network with two nodes, shown in *Fig. 1*. We want to send two commodity t_1 and t_2 from node 1 to node 2. There exist three conveyances between these two nodes, and each conveyance has a total capacity e and furthermore, each conveyance has a capacity for each commodity as e_{t_1} and e_{t_2}. Note that we cannot send commodities more of the total capacity e, while we can have $e_{t_1} + e_{t_2} \geq e$.

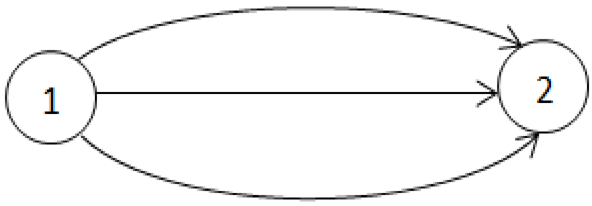

Fig. 1. Network representing FMMMCF.

Similar to [8], [9] we categorize the nodes as follows.

Purely source node: those nodes S in which there exists some node S' such that the product may be supplied from S to S' while there does not exist any node S'' to transport product from S'' to S. The set of all such nodes is denoted by N_{PS}.

Purely destination node: those nodes D in which there does not exist any node D' such that the product may be supplied from D to D' while there exists some node D'' to transport product from D'' to D. The set of all such nodes is denoted by N_{PD}.

Intermediate node: the intermediate nodes are another part of a network. In the following, we sort the types of these nodes.

I. Those nodes S have some quantity of the product for supplying to other nodes, and also, there exist some nodes such that some amount of the product is transported from those nodes to node S. Such nodes are called source nodes, and the set of all such nodes is dented by N_S.

II. Those nodes D which require some quantity of the product, and also there exist some nodes such that the product is supplied from node D to those nodes. Such nodes are called destination nodes, and the set of all such nodes is dented by N_D.

III. Those nodes T which neither any quantity of the product is available at them to transship, nor any quantity of the products is required, and all quantity of the product transferred from some nodes to node T is supplied from T to some other nodes. Such nodes are called transition nodes, and the set of all such nodes is dented by N_T.

In the following, we list notations we use to represent our model.

I. $\tilde{a}_i^t$: the fuzzy availability of the product t at ith purely source node.

II. $\tilde{a}_i^{t\prime}$: the fuzzy availability of the product t at ith source node.

III. $\tilde{b}_j^t$: the fuzzy demand of the product t at jth purely destination node.

IV. $\tilde{b}_j^{t\prime}$: the fuzzy demand of the product t at jth destination node.

V. $\tilde{e}_k^t$: the fuzzy capacity of the kth conveyance for transfer of the product t.

VI. $\tilde{e}_k$: the total fuzzy capacity of the kth conveyance.

VII. $\tilde{c}_{ijk}^{tl}$: the fuzzy penalty per unit of flow t from ith (purely) source to jth (purely) destination by means of the kth conveyance in the lth objective function.

VIII. $\tilde{x}_{ijk}^t$: the fuzzy quantity of the product t that should be transported from ith node to jth node by means of the kth conveyance to minimize all objective functions.

IX. S_C: the set of all available conveyances.

We assume that $\tilde{a}_i^t$, $\tilde{a}_i^{t\prime}$,$\tilde{b}_j^t$, $\tilde{b}_j^{t\prime}$, $\tilde{e}_k^t$, $\tilde{e}_k$ are non-negative LR flat fuzzy numbers. We also assume that there are L objective functions and T commodities. With these notations, a FMMMCF problem can be formulated into the following fuzzy multiobjective linear programming problem:

$$\text{Minimum} \quad \sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} (\tilde{c}_{ijk}^{tl} \otimes \tilde{x}_{ijk}^t) \quad l = 1, \dots, L, \tag{1}$$

Subject to

$$\sum_{j:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk}^t \lessapprox \tilde{a}_i^t, \quad i \in N_{PS}, t = 1, \dots, T,$$
$$\sum_{j:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk}^t \lessapprox \sum_{j:(j,i)\in A} \sum_{k\in S_C} \tilde{x}_{jik}^t \oplus \tilde{a}_i^{t\prime}, \quad i \in N_S, t = 1, \dots, T,$$
$$\sum_{i:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk}^t \gtrapprox \tilde{b}_j^t, \quad j \in N_{PD}, t = 1, \dots, T,$$
$$\sum_{i:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk}^t \gtrapprox \sum_{i:(j,i)\in A} \sum_{k\in S_C} \tilde{x}_{jik}^t \oplus \tilde{b}_j^{t\prime}, \quad j \in N_D, t = 1, \dots, T,$$
$$\sum_{j:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk}^t = \sum_{j:(j,i)\in A} \sum_{k\in S_C} \tilde{x}_{jik}^t, \quad i \in N_T, t = 1, \dots, T,$$
$$\sum_{t=1}^{T} \sum_{j:(i,j)\in A} \tilde{x}_{ijk}^t \lessapprox \tilde{e}_k, \quad k \in S_C,$$

$$\tilde{x}^t_{ijk} \lessapprox \tilde{e}^t_k, \quad k \in S_C, (i,j) \in A, t = 1, \dots, T.$$

where $\tilde{x}^t_{ijk}$ is a non-negative LR flat fuzzy number for all $(i,j) \in A$ and $k \in S_C$.

4 | Proposed Method

Almost in all available algorithms for MCF problems, we must examine whether the problem is balanced or unbalanced and convert an unbalanced one to a balanced one with some modifications. This process may be expensive, so solving our model without this assumption is better. In our model, there are some equalities and inequalities; therefore, with a generalization of the proposed algorithm in Subsection 5.4.2 of [23] and using existing methods [24] to solve multiobjective linear programming, we can obtain the optimal compromise solution for our model. We recall the definition of a fuzzy efficient solution from the literature.

Definition 5. A fuzzy, feasible solution $\tilde{x} = \{\tilde{x}^t_{ijk}\}$ is said to be a fuzzy efficient solution of the fully fuzzy multiobjective SMMMCF problem if there is no other fuzzy feasible solution $\tilde{x}' = \{\tilde{x}^{t\prime}_{ijk}\}$ such that

$$\sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} \Re(\tilde{c}^{tl}_{ijk} \otimes \tilde{x}^{t\prime}_{ijk}) \leq \sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} \Re(\tilde{c}^{tl}_{ijk} \otimes \tilde{x}^{t}_{ijk}),$$

for all $l \in \{1, \dots, L\}$, and

$$\sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} \Re(\tilde{c}^{tl}_{ijk} \otimes \tilde{x}^{t\prime}_{ijk}) < \sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} \Re(\tilde{c}^{tl}_{ijk} \otimes \tilde{x}^{t}_{ijk}),$$

for at least one $l \in \{1, \dots, L\}$.

Note that, for real-world problems, we do not need to obtain the set of all fuzzy efficient solutions. It is sufficient to compute a fuzzy optimal compromise solution. A fuzzy optimal compromise solution of the FMMMCF problem is a feasible solution that the decision maker prefers to all other solutions. We accept that a fuzzy optimal compromise solution must be a fuzzy efficient solution.

Step 1. Assume that $\tilde{c}^{tl}_{ijk} = (p^{tl}_{ijk}, q^{tl}_{ijk}, \alpha^{tl}_{ijk}, \beta^{tl}_{ijk})_{LR}$, $\tilde{x}^t_{ijk} = (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR}$, $\tilde{a}^t_i = (r^t_i, s^t_i, \varepsilon^t_i, \zeta^t_i)_{LR}$, $\tilde{a}^{t\prime}_i = (r^{t\prime}_i, s^{t\prime}_i, \varepsilon^{t\prime}_i, \zeta^{t\prime}_i)_{LR}$, $\tilde{b}^t_j = (v^t_j, w^t_j, \eta^t_j, \theta^t_j)_{LR}$, $\tilde{b}^{t\prime}_j = (v^{t\prime}_j, w^{t\prime}_j, \eta^{t\prime}_j, \theta^{t\prime}_j)_{LR}$, $\tilde{e}_k = (g_k, h_k, \lambda_k, \mu_k)_{LR}$, and $\tilde{e}^t_k = (g^t_k, h^t_k, \lambda^t_k, \mu^t_k)_{LR}$ .Therefore, *Problem (1)* can be written as

$$\text{Minimum} \sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} (p^{tl}_{ijk}, q^{tl}_{ijk}, \alpha^{tl}_{ijk}, \beta^{tl}_{ijk})_{LR} \otimes (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR} = 1, \dots, L, \quad (2)$$

Subject to

$$\sum_{j:(i,j)\in A} \sum_{k\in S_C} (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR} \lessapprox (r^t_i, s^t_i, \varepsilon^t_i, \zeta^t_i)_{LR}, \quad i \in N_{PS}, t = 1, \dots, T,$$

$$\sum_{J:(I,j)\in A} \sum_{k\in S_C} (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR} \lessapprox \sum_{j:(j,i)\in A} \sum_{k\in S_C} (y^t_{jik}, z^t_{jik}, \gamma^t_{jik}, \delta^t_{jik})_{LR} \oplus \left(r^{t'}_i, s^{t'}_i, \varepsilon^{t'}_i, \zeta^{t'}_i\right)_{LR}, \quad i \in N_S, t = 1, \dots, T,$$

$$\sum_{i:(i,j)\in A} \sum_{k\in S_C} (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR} \gtrapprox (v^t_j, w^t_j, \eta^t_j, \theta^t_j), \quad j \in N_{PD}, t = 1, \dots, T,$$

$$\sum_{i:(i,j)\in A} \sum_{k\in S_C} (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR} \gtrapprox \sum_{i:(j,i)\in A} \sum_{k\in S_C} (y^t_{jik}, z^t_{jik}, \gamma^t_{jik}, \delta^t_{jik})_{LR} \oplus \left(v^{t'}_j, w^{t'}_j, \eta^{t'}_j, \theta^{t'}_j\right)_{LR}, \quad j \in N_D, t = 1, \dots, T,$$

$$\sum_{j:(i,j)\in A} \sum_{k\in S_C} (y^t_{ijk}, z^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk})_{LR} = \sum_{j:(j,i)\in A} \sum_{k\in S_C} (y^t_{jik}, z^t_{jik}, \gamma^t_{jik}, \delta^t_{jik})_{LR}, \quad i \in N_T, t = 1, \dots, T,$$

$$\sum_{t=1}^{T}\sum_{j:(i,j)\in A}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\precsim(g_k,h_k,\lambda_k,\mu_k)_{LR},\quad k\in S_C,$$
$$(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\precsim(g_k^{t},h_k^{t},\lambda_k^{t},\mu_k^{t})_{LR},\quad k\in S_C,(i,j)\in A,t=1,\dots,T.$$

where $(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}$ is a non-negative LR flat fuzzy number for all $k\in S_C,(i,j)\in A,t=1,\dots,T.$

Step 2. Assume that

$$(p_{ijk}^{tl},q_{ijk}^{tl},\alpha_{ijk}^{tl},\beta_{ijk}^{tl})_{LR}\otimes(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}=(o_{ijk}^{tl},u_{ijk}^{tl},\varphi_{ijk}^{tl},\tau_{ijk}^{tl})_{LR},$$
$$(y_{jik}^{t},z_{jik}^{t},\gamma_{jik}^{t},\delta_{jik}^{t})_{LR}\oplus(r_i^{t\prime},s_i^{t\prime},\varepsilon_i^{t\prime},\zeta_i^{t\prime})_{LR}=(m_{jik}^{t},n_{jik}^{t},\pi_{jik}^{t},\sigma_{jik}^{t})_{LR},$$

and

$$(y_{jik}^{t},z_{jik}^{t},\gamma_{jik}^{t},\delta_{jik}^{t})_{LR}\oplus(v_j^{t\prime},w_j^{t\prime},\eta_j^{t\prime},\theta_j^{t\prime})_{LR}=(m_{jik}^{t\prime},n_{jik}^{t\prime},\pi_{jik}^{t\prime},\sigma_{jik}^{t\prime})_{LR}.$$

With these notations, *Problem (2)* can be written as

$$\text{Minimum}\quad\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}(o_{ijk}^{tl},u_{ijk}^{tl},\varphi_{ijk}^{tl},\tau_{ijk}^{tl})_{LR},\quad l=1,\dots,L,\qquad\textbf{(3)}$$

Subject to

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\precsim(r_i^{t},s_i^{t},\varepsilon_i^{t},\zeta_i^{t})_{LR},\quad i\in N_{PS},t=1,\dots,T,$$
$$\textstyle\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\precsim(m_{jik}^{t},n_{jik}^{t},\pi_{jik}^{t},\sigma_{jik}^{t})_{LR},\quad i\in N_S,t=1,\dots,T,$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\succsim(v_j^{t},w_j^{t},\eta_j^{t},\theta_j^{t}),\quad j\in N_{PD},t=1,\dots,T,$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\succsim\sum_{i:(j,i)\in A}\sum_{k\in S_C}(m_{jik}^{t\prime},n_{jik}^{t\prime},\pi_{jik}^{t\prime},\sigma_{jik}^{t\prime})_{LR},\quad j\in N_D,t=1,\dots,T,$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}=\sum_{j:(j,i)\in A}\sum_{k\in S_C}(y_{jik}^{t},z_{jik}^{t},\gamma_{jik}^{t},\delta_{jik}^{t})_{LR},$$
$$i\in N_T,t=1,\dots,T,$$
$$\sum_{t=1}^{T}\sum_{j:(i,j)\in A}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\precsim(g_k,h_k,\lambda_k,\mu_k)_{LR},\quad k\in S_C,$$
$$(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\precsim(g_k^{t},h_k^{t},\lambda_k^{t},\mu_k^{t})_{LR},\quad k\in S_C,(i,j)\in A,t=1,\dots,T.$$

where $(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}$ is a non-negative LR flat fuzzy number for all $k\in S_C,(i,j)\in A,t=1,\dots,T.$

Step 3. Using rank function $\Re$, we solve the following problem:

$$\text{Minimum}\quad\Re\left(\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}(o_{ijk}^{tl},u_{ijk}^{tl},\varphi_{ijk}^{tl},\tau_{ijk}^{tl})_{LR}\right)\quad l=1,\dots,L,\qquad\textbf{(4)}$$

Subject to

$$\Re\left(\textstyle\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\right)\le\Re(r_i^{t},s_i^{t},\varepsilon_i^{t},\zeta_i^{t})_{LR},\quad i\in N_{PS},t=1,\dots,T,$$
$$\Re\left(\textstyle\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\right)\le\Re(m_{jik}^{t},n_{jik}^{t},\pi_{jik}^{t},\sigma_{jik}^{t})_{LR},\quad i\in N_S,t=1,\dots,T,$$
$$\Re\left(\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^{t},z_{ijk}^{t},\gamma_{ijk}^{t},\delta_{ijk}^{t})_{LR}\right)\ge\Re(v_j^{t},w_j^{t},\eta_j^{t},\theta_j^{t}),\quad j\in N_{PD},t=1,\dots,T,$$

$$\Re\left(\sum_{i:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR}\right) \geq \Re\left(\sum_{i:(j,i)\in A}\sum_{k\in S_C}\left(m_{jik}^{t'}, n_{jik}^{t'}, \pi_{jik}^{t'}, \sigma_{jik}^{t'}\right)_{LR}\right),$$

$j \in N_D, t = 1, \dots, T,$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} y_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} y_{jik}^t, \quad i \in N_T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} z_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} z_{jik}^t, \quad i \in N_T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \gamma_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \gamma_{jik}^t, \quad i \in N_T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \delta_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \delta_{jik}^t, \quad i \in N_T,$$

$$\Re\left(\sum_{t=1}^{T}\sum_{j:(i,j)\in A}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR}\right) \leq \Re\left(g_k, h_k, \lambda_k, \mu_k\right)_{LR} \quad k \in S_C,$$

$$\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \leq \Re\left(g_k^t, h_k^t, \lambda_k^t, \mu_k^t\right)_{LR}, \quad k \in S_C, (i,j) \in A, t = 1, \dots, T,$$

$$y_{ijk}^t - \gamma_{ijk}^t, z_{ijk}^t - y_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t \geq 0, \quad k \in S_C, (i,j) \in A, t = 1, \dots, T.$$

Step 4. Concerning the linear property of the rank *Function (4)* can be written as:

$$\text{Minimum} \quad \sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C} \Re\left(o_{ijk}^{tl}, u_{ijk}^{tl}, \varphi_{ijk}^{tl}, \tau_{ijk}^{tl}\right)_{LR} \quad l = 1, \dots, L, \tag{5}$$

Subject to

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \leq \Re\left(r_i^t, s_i^t, \varepsilon_i^t, \zeta_i^t\right)_{LR}, \quad i \in N_{PS}, t = 1, \dots, T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \leq \Re\left(m_{jik}^t, n_{jik}^t, \pi_{jik}^t, \sigma_{jik}^t\right)_{LR}, \quad i \in N_S, t = 1, \dots, T,$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} \Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \geq \Re\left(v_j^t, w_j^t, \eta_j^t, \theta_j^t\right), \quad j \in N_{PD}, t = 1, \dots, T,$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} \Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \geq \sum_{i:(j,i)\in A}\sum_{k\in S_C} \Re\left(m_{jik}^{t'}, n_{jik}^{t'}, \pi_{jik}^{t'}, \sigma_{jik}^{t'}\right)_{LR},$$

$j \in N_D, t = 1, \dots, T,$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} y_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} y_{jik}^t, \quad i \in N_T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} z_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} z_{jik}^t, \quad i \in N_T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \gamma_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \gamma_{jik}^t, \quad i \in N_T,$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \delta_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \delta_{jik}^t, \quad i \in N_T,$$

$$\sum_{t=1}^{T}\sum_{j:(i,j)\in A} \Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \leq \Re\left(g_k, h_k, \lambda_k, \mu_k\right)_{LR}, \quad k \in S_C,$$

$$\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \leq \Re\left(g_k^t, h_k^t, \lambda_k^t, \mu_k^t\right)_{LR}, \quad k \in S_C, (i,j) \in A, t = 1, \dots, T,$$

$y_{ijk}^{t} - \gamma_{ijk}^{t}, z_{ijk}^{t} - y_{ijk}^{t}, \gamma_{ijk}^{t}, \delta_{ijk}^{t} \geq 0, \quad k \in S_C, (i,j) \in A, t = 1, \ldots, T.$

Step 5. With solving the crisp programming *Problem (5)*, find the optimal compromise solution $x_{ijk}^{t*} = \left(y_{ijk}^{t*}, z_{ijk}^{t*}, \gamma_{ijk}^{t*}, \delta_{ijk}^{t*}\right)_{LR}$.

Step 6. Find the fuzzy optimal value of each objective function by putting the values of

$x_{ijk}^{t*} = \left(y_{ijk}^{t*}, z_{ijk}^{t*}, \gamma_{ijk}^{t*}, \delta_{ijk}^{t*}\right)_{LR}$ in $\sum_{t=1}^{T} \sum_{(i,j)\in A} \sum_{k\in S_C} (\tilde{c}_{ijk}^{tl} \otimes \tilde{x}_{ijk}^{t})$.

5 | Illustrative Example

In this section, we illustrate our method with an example.

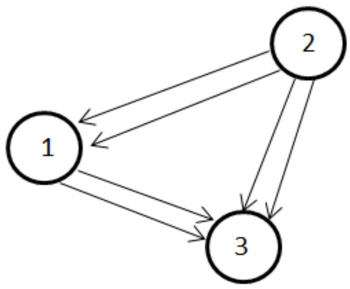

Fig. 2. Network representing Example 1.

Example 1. Consider the network *Fig. 2* with the following data. Fuzzy penalty for 1st objective function to transport commodity 1st

$c_{131}^{11} = (3,4,2,2)_{LR}, \quad c_{132}^{11} = (2,3,1,2)_{LR}.$

$c_{211}^{11} = (4,5,3,3)_{LR}, \quad c_{212}^{11} = (5,6,3,3)_{LR}.$

$c_{231}^{11} = (5,7,4,3)_{LR}, \quad c_{232}^{11} = (3,4,2,3)_{LR}.$

Fuzzy penalty for 2nd objective function to transport commodity 1st

$c_{131}^{12} = (4,6,3,3)_{LR}, \quad c_{132}^{12} = (3,4,2,2)_{LR}.$

$c_{211}^{12} = (5,6,4,3)_{LR}, \quad c_{212}^{12} = (6,7,4,4)_{LR}.$

$c_{231}^{12} = (6,8,4,4)_{LR}, \quad c_{232}^{12} = (4,5,2,3)_{LR}.$

Fuzzy penalty for 1st objective function to transport commodity 2nd

$c_{131}^{21} = (2,3,1,1)_{LR}, \quad c_{132}^{21} = (2,4,1,2)_{LR}.$

$c_{211}^{21} = (4,5,3,2)_{LR}, \quad c_{212}^{21} = (2,3,2,2)_{LR}.$

$c_{231}^{21} = (4,6,3,3)_{LR}, \quad c_{232}^{21} = (2,3,1,2)_{LR}.$

Fuzzy penalty for 2nd objective function to transport commodity 2nd

$c_{131}^{22} = (3,4,2,1)_{LR}, \quad c_{132}^{22} = (3,4,2,2)_{LR}.$

$c_{211}^{22} = (5,7,4,3)_{LR}, \quad c_{212}^{22} = (5,6,3,3)_{LR}.$

$c_{231}^{22} = (5,6,4,3)_{LR}$, $c_{232}^{22} = (2,3,1,1)_{LR}$.

Fuzzy availability of the commodity 1st at source node 1 and purely source node 2

$a_1^{1\prime} = (30,40,20,10)_{LR}$, $a_2^1 = (30,40,20,20)_{LR}$.

Fuzzy availability of the commodity 2nd at source node 1 and purely source node 2

$a_1^{2\prime} = (40,60,20,30)_{LR}$, $a_2^2 = (40,50,30,30)_{LR}$.

Fuzzy demand of the commodity 1st at purely destination node 3

$b_3^1 = (30,50,20,30)_{LR}$.

Fuzzy demand of the commodity 2nd at purely destination node 3

$b_3^2 = (40,50,30,30)_{LR}$.

Fuzzy capacity of the 1st conveyance for transfer of the commodities 1st and 2nd

$e_1^1 = (60,70,40,30)_{LR}$, $e_1^2 = (60,70,30,30)_{LR}$.

Fuzzy capacity of the 2nd conveyance for transfer of the commodities 1st and 2nd

$e_2^1 = (60,70,30,30)_{LR}$, $e_2^2 = (60,70,20,20)_{LR}$.

The total fuzzy capacity of the 1st and 2nd conveyances

$e_1 = (70,70,30,30)_{LR}$, $e_2 = (70,80,20,20)_{LR}$.

We assume that $L(x) = R(x) = \max\{0, 1 - x^4\}$. Therefore, for a fuzzy number $\tilde{a} = (m,n,\alpha,\beta)$, $\Re(\tilde{a}) = \frac{1}{2}(m+n) + \frac{4}{15}(\beta - \alpha)$ (see Remark 1 in [9]).

The model will be as follows:

$$
\begin{aligned}
&\text{Minimum } (3,4,2,2)_{LR} \otimes x_{131}^1 \oplus (2,3,1,2)_{LR} \otimes x_{132}^1 \oplus (4,5,3,3)_{LR} \otimes x_{211}^1 \\
&\oplus (5,6,3,3)_{LR} \otimes x_{212}^1 \oplus (5,7,4,3)_{LR} \otimes x_{231}^1 \oplus (3,4,2,3)_{LR} \otimes x_{232}^1 \\
&\oplus (2,3,1,1)_{LR} \otimes x_{131}^2 \oplus (2,4,1,2)_{LR} \otimes x_{132}^2 \oplus (4,5,3,2)_{LR} \otimes x_{211}^2 \\
&\oplus (2,3,2,2)_{LR} \otimes x_{212}^2 \oplus (4,6,3,3)_{LR} \otimes x_{231}^2 \oplus (2,3,1,2)_{LR} \otimes x_{232}^2, \\
&\text{Minimum } (4,6,3,3)_{LR} \otimes x_{131}^1 \oplus (3,4,2,2)_{LR} \otimes x_{132}^1 \oplus (5,6,4,3)_{LR} \otimes x_{211}^1 \\
&\oplus (6,7,4,4)_{LR} \otimes x_{212}^1 \oplus (6,8,4,4)_{LR} \otimes x_{231}^1 \oplus (4,5,2,3)_{LR} \otimes x_{232}^1 \\
&\oplus (3,4,2,1)_{LR} \otimes x_{131}^2 \oplus (3,4,2,2)_{LR} \otimes x_{132}^2 \oplus (5,7,4,3)_{LR} \otimes x_{211}^2 \oplus \\
&(5,6,3,3)_{LR} \otimes x_{212}^2 \oplus (5,6,4,3)_{LR} \otimes x_{231}^2 \oplus (2,3,1,1)_{LR} \otimes x_{232}^2, \\
&\text{Subject to} \\
&x_{211}^1 \oplus x_{212}^1 \oplus x_{231}^1 \oplus x_{232}^1 \precsim (30,40,20,20)_{LR}, \\
&x_{211}^2 \oplus x_{212}^2 \oplus x_{231}^1 \oplus x_{232}^2 \precsim (40,50,30,30)_{LR}, \\
&x_{131}^1 \oplus x_{132}^1 \precsim x_{211}^1 \oplus x_{212}^1 \oplus (30,40,20,10)_{LR}, \\
&x_{131}^2 \oplus x_{132}^2 \precsim x_{211}^2 \oplus x_{212}^2 \oplus (40,60,20,30)_{LR}, \\
&x_{131}^1 \oplus x_{132}^1 \oplus x_{231}^1 \oplus x_{232}^1 \succsim (30,50,20,30)_{LR}, \\
&x_{131}^2 \oplus x_{132}^2 \oplus x_{231}^2 \oplus x_{232}^2 \succsim (40,50,30,30)_{LR}, \\
&x_{211}^1 \oplus x_{231}^1 \oplus x_{131}^1 \oplus x_{211}^2 \oplus x_{231}^2 \oplus x_{131}^2 \precsim (70,70,30,30)_{LR},
\end{aligned}
\tag{6}
$$

$x^1_{211} \lesssim (60, 70, 40, 30)_{LR}$,
$x^1_{212} \lesssim (60, 70, 30, 30)_{LR}$,
$x^1_{231} \lesssim (60, 70, 40, 30)_{LR}$,
$x^1_{232} \lesssim (60, 70, 30, 30)_{LR}$,
$x^2_{211} \lesssim (60, 70, 30, 30)_{LR}$,
$x^2_{212} \lesssim (60, 70, 20, 20)_{LR}$,
$x^2_{231} \lesssim (60, 70, 30, 30)_{LR}$,
$x^2_{231} \lesssim (60, 70, 20, 20)_{LR}$.

And x^t_{ijk} $(i = j = 1, 2, 3, k, t = 1, 2)$ is a non-negative LR flat fuzzy number. Concerning *Steps 3* and *4* in Section 4, the fuzzy optimal solution can be obtained by solving the following problem:

$$\begin{aligned}
&\text{Minimum } \tfrac{1}{30}(61\, y^1_{131} + 76\, z^1_{131} + 8\, \gamma^1_{131} + 48\, \delta^1_{131} + 38\, y^1_{132} + 61\, z^1_{132} + 8\, \gamma^1_{132} + \\
&40\, \delta^1_{132} + 84\, y^1_{211} + 99\, z^1_{211} + 8\, \gamma^1_{211} + 64\, \delta^1_{211} + 99\, y^1_{212} + 114\, z^1_{212} + 16\, \gamma^1_{212} + \\
&72\, \delta^1_{212} + 107 y^1_{231} + 129\, z^1_{231} + 8\, \gamma^1_{231} + 80\, \delta^1_{231} + 61 y^1_{232} + 84\, z^1_{232} + 8\, \gamma^1_{232} + 56\, \delta^1_{232} + \\
&38\, y^2_{131} + 53\, z^2_{131} + 8\, \gamma^2_{131} + 32\, \delta^2_{131} + 38\, y^2_{132} + 76\, z^2_{132} + 8\, \gamma^2_{132} + 48\, \delta^2_{132} + 84\, y^2_{211} + \\
&91\, z^2_{211} + 8\, \gamma^2_{211} + 56\, \delta^2_{211} + 46 y^2_{212} + 61\, z^2_{212} + 0\, \gamma^2_{212} + 40\, \delta^2_{212} + 84 y^2_{231} + 114\, z^2_{231} + \\
&8\, \gamma^2_{231} + 72\, \delta^2_{231} + 38 y^2_{232} + 61\, z^2_{232} + 8\, \gamma^2_{232} + 40\, \delta^2_{232}),
\end{aligned} \tag{7}$$

$$\begin{aligned}
\text{Minimum } \frac{1}{30}(&84\, y^1_{131} + 114\, z^1_{131} + 8\, \gamma^1_{131} + 72\, \delta^1_{131} + 61\, y^1_{132} + 76\, z^1_{132} + 8\, \gamma^1_{132} \\
&+ 48\, \delta^1_{132} + 107\, y^1_{211} + 114\, z^1_{211} + 8\, \gamma^1_{211} + 72\, \delta^1_{211} + 122\, y^1_{212} \\
&+ 137\, z^1_{212} + 16\, \gamma^1_{212} + 88\, \delta^1_{212} + 122 y^1_{231} + 152\, z^1_{231} + 16\, \gamma^1_{231} \\
&+ 96\, \delta^1_{231} + 76 y^1_{232} + 99\, z^1_{232} + 16\, \gamma^1_{232} + 64\, \delta^1_{232} + 61\, y^2_{131} \\
&+ 68\, z^2_{131} + 8\, \gamma^2_{131} + 40\, \delta^2_{131} + 61\, y^2_{132} + 76\, z^2_{132} + 8\, \gamma^2_{132} + 48\, \delta^2_{132} \\
&+ 107\, y^2_{211} + 129\, z^2_{211} + 8\, \gamma^2_{211} + 80\, \delta^2_{211} + 99 y^2_{212} + 114\, z^2_{212} \\
&+ 16\, \gamma^2_{212} + 72\, \delta^2_{212} + 107 y^2_{231} + 114\, z^2_{231} + 8\, \gamma^2_{231} + 72\, \delta^2_{231} \\
&+ 38 y^2_{232} + 53\, z^2_{232} + 8\, \gamma^2_{232} + 32\, \delta^2_{232}),
\end{aligned}$$

Subject to

$$\begin{aligned}
&\frac{1}{2}(y^1_{211} + y^1_{212} + y^1_{231} + y^1_{232} + z^1_{211} + z^1_{212} + z^1_{231} + z^1_{232}) \\
&\qquad + \frac{4}{15}(\delta^1_{211} + \delta^1_{212} + \delta^1_{231} + \delta^1_{232} - \gamma^1_{211} - \gamma^1_{212} - \gamma^1_{231} - \gamma^1_{232}) \le 35,
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}(y^2_{211} + y^2_{212} + y^2_{231} + y^2_{232} + z^2_{211} + z^2_{212} + z^2_{231} + z^2_{232}) \\
&\qquad + \frac{4}{15}(\delta^2_{211} + \delta^2_{212} + \delta^2_{231} + \delta^2_{232} - \gamma^2_{211} - \gamma^2_{212} - \gamma^2_{231} - \gamma^2_{232}) \le 45,
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}(y^1_{131} + y^1_{132} + z^1_{131} + z^1_{132}) + \frac{4}{15}(\delta^1_{131} + \delta^1_{132} - \gamma^1_{131} - \gamma^1_{132}) \\
&\qquad \le \frac{1}{2}(70 + y^1_{211} + y^1_{212} + z^1_{211} + z^1_{212}) \\
&\qquad + \frac{4}{15}(\delta^1_{211} + \delta^1_{212} - \gamma^1_{211} - \gamma^1_{212} - 10),
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{2}(y^2_{131} + y^2_{132} + z^2_{131} + z^2_{132}) + \tfrac{4}{15}(\delta^2_{131} + \delta^2_{132} - \gamma^2_{131} - \gamma^2_{132}) \le \tfrac{1}{2}(100 + y^2_{211} + y^2_{212} + \\
&z^2_{211} + z^2_{212}) + \tfrac{4}{15}(\delta^2_{211} + \delta^2_{212} - \gamma^2_{211} - \gamma^2_{212} + 10),
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{2}(y^1_{131} + y^1_{132} + y^1_{231} + y^1_{232} + z^1_{131} + z^1_{132} + z^1_{231} + z^1_{232}) + \tfrac{4}{15}(\delta^1_{131} + \delta^1_{132} + \delta^1_{231} + \\
&\delta^1_{232} - \gamma^1_{131} - \gamma^1_{132} - \gamma^1_{231} - \gamma^1_{232}) \ge \tfrac{128}{3},
\end{aligned}$$

$$\frac{1}{2}(y_{131}^2 + y_{132}^2 + y_{231}^2 + y_{232}^2 + z_{131}^2 + z_{132}^2 + z_{231}^2 + z_{232}^2)$$
$$+ \frac{4}{15}(\delta_{131}^2 + \delta_{132}^2 + \delta_{231}^2 + \delta_{232}^2 - \gamma_{131}^2 - \gamma_{132}^2 - \gamma_{231}^2 - \gamma_{232}^2) \geq 45,$$
$$\frac{1}{2}(y_{211}^1 + y_{231}^1 + y_{131}^1 + y_{211}^2 + y_{231}^2 + y_{131}^1 + z_{211}^1 + z_{231}^1 + z_{131}^1 + z_{211}^2 + z_{231}^2 + z_{131}^2)$$
$$+ \frac{4}{15}(\delta_{211}^1 + \delta_{231}^1 + \delta_{131}^1 + \delta_{211}^2 + \delta_{231}^2 + \delta_{131}^2 - \gamma_{211}^1 - \gamma_{231}^1 - \gamma_{131}^1$$
$$- \gamma_{211}^2 - \gamma_{231}^2 - \gamma_{131}^2) \leq 70,$$
$$\frac{1}{2}(y_{211}^1 + z_{211}^1) + \frac{4}{15}(\delta_{211}^1 - \gamma_{211}^1) \leq \frac{187}{3},$$
$$\frac{1}{2}(y_{212}^1 + z_{212}^1) + \frac{4}{15}(\delta_{212}^1 - \gamma_{212}^1) \leq 65,$$
$$\frac{1}{2}(y_{231}^1 + z_{231}^1) + \frac{4}{15}(\delta_{231}^1 - \gamma_{231}^1) \leq \frac{187}{3},$$
$$\frac{1}{2}(y_{232}^1 + z_{232}^1) + \frac{4}{15}(\delta_{232}^1 - \gamma_{232}^1) \leq 65,$$
$$\frac{1}{2}(y_{211}^2 + z_{211}^2) + \frac{4}{15}(\delta_{211}^2 - \gamma_{211}^2) \leq 65,$$
$$\frac{1}{2}(y_{212}^2 + z_{212}^2) + \frac{4}{15}(\delta_{212}^2 - \gamma_{212}^2) \leq 65,$$
$$\frac{1}{2}(y_{231}^2 + z_{231}^2) + \frac{4}{15}(\delta_{231}^2 - \gamma_{231}^2) \leq 65,$$
$$\frac{1}{2}(y_{232}^2 + z_{232}^2) + \frac{4}{15}(\delta_{232}^2 - \gamma_{232}^2) \leq 65,$$
$$y_{ijk}^t - \gamma_{ijk}^t, z_{ijk}^t - y_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t \geq 0, \quad k \in S_C, (i,j) \in A, t = 1, \dots, T.$$

By solving this problem using the weighted sum method [24], we have

y_{231}^1	0.0415	y_{232}^1	0.0103	y_{211}^1	0.0627
z_{231}^1	0.0198	z_{232}^1	0.0103	z_{211}^1	0.0979
γ_{231}^1	0	γ_{232}^1	0.6951	γ_{211}^1	0
δ_{231}^1	0	δ_{232}^1	0	δ_{211}^1	0

y_{212}^1	0.1259	y_{131}^1	0.0322	y_{132}^1	0.0481
z_{212}^1	0.1259	z_{331}^1	0.0322	z_{132}^1	0.2467
γ_{212}^1	0	γ_{131}^1	0	γ_{132}^1	0
δ_{212}^1	0	δ_{131}^1	0.1873	δ_{132}^1	0

y_{231}^2	0.0208	y_{232}^2	0.0437	y_{211}^2	0.0474
z_{231}^2	0.1824	z_{232}^2	0.0437	z_{211}^2	0.087
γ_{231}^2	0	γ_{232}^2	0	γ_{211}^2	0.0215
δ_{231}^2	0	δ_{232}^2	0	δ_{211}^2	0

y_{212}^2	-0.1491	y_{131}^2	-0.3316	y_{132}^2	-0.4967
z_{212}^2	-0.1491	z_{131}^2	-0.3316	z_{132}^2	-1.0268
γ_{212}^2	0	γ_{131}^2	0.3438	γ_{132}^2	0
δ_{212}^2	0	δ_{131}^2	0	δ_{132}^2	0

6 | Conclusion

In this paper, we introduced a new model for fully fuzzy multiobjective multicommodity minimal cost flow problems in which there are several commodities to transport from sources to destinations, and there is more than one conveyance for this transporting. We also assume that each commodity has distinct capacities in each conveyance. We proposed a method for solving this problem without considering a balanced version of that. Our method can also be considered a generalization of some methods for solving fuzzy multiobjective methods in the presence of equalities and fuzzy inequalities.

Author Contribution

Conceptualization, M. Gh.; Methodology, B. Kh., Software, Validation, formal analysis, and investigation, V. I..; writing-creating the initial design, M. Gh.; writing-reviewing and editing, B. kh. All authors have read and agreed to the published version of the manuscript.

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All the data are available in this paper.

Conflicts of Interest

The authors declare no conflict of interest.

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