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A Unified Hypergraph- and SuperHyperGraph-Based Framework for Food Web Extension: From Classical Food Webs to SuperHyperWebs in Agricultural and Ecological Systems

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Abstract

Research on the applications of graph theory has also been conducted in areas such as Agricultural and Ecological Systems. Hypergraphs generalize graphs by allowing hyperedges to join any number of vertices, while superhypergraphs further extend this idea by layering iterated powersets to capture hierarchical, self-referential connections. A food web models an ecosystem as a directed graph whose nodes are species and whose edges represent predator–prey interactions. In this paper, we introduce two novel extensions of classical food webs: the *Food HyperWeb*, which encodes each predator's entire prey set as a hyperedge, and the *Food n-SuperHyperWeb*, which embeds multilevel trophic relationships within an n-fold superhypergraph structure. We provide formal definitions, establish their foundational properties, and present illustrative examples demonstrating their effectiveness for agricultural and ecological network analysis.

Keywords: Superhypergraph, Hypergraph, Food web, Food hyperweb, Food n-superhyperweb

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1| Preliminaries

This section introduces the fundamental concepts and definitions that underpin the discussions in this paper. Throughout, all sets are assumed to be finite.

1.1| Power Set and the n-th Iterated Power Set

For a given set S, its power set is the family of all subsets of S, including both the empty set and S itself. The n-th iterated power set of S is obtained by repeatedly applying the power set operation n times, beginning with S [1, 2].

Definition 1.1 (Universal Set)**.** Let U denote a set that contains every element under discussion. Throughout, any set S is assumed to satisfy $S \subseteq U$.

Definition 1.2 (Base Set)**.** A *base set* S is defined as a subset of U that serves as the starting point for further constructions, such as power sets and hyperstructures.

Definition 1.3 (Power Set)**.** [3] The *power set* of a set S, written $\mathcal{P}(S)$, is given by

$$\mathcal{P}(S) = \{\, X \mid X \subseteq S \,\}.$$

Definition 1.4 (Iterated Power Set)**.** [4, 5, 6, 7] For each integer $n \geq 1$, the n-fold iterated power set of S is defined recursively as

$$\mathcal{P}^1(S) = \mathcal{P}(S),$$
$$\mathcal{P}^{k+1}(S) = \mathcal{P}\big(\mathcal{P}^k(S)\big), \quad k \geq 1.$$

Equivalently, one may denote $P_n(S) = \mathcal{P}^n(S)$.

Definition 1.5 (Nonempty Iterated Power Set)**.** [4, 8] The *nonempty iterated power set* is given by

$$\mathcal{P}_1^*(S) = \mathcal{P}(S) \setminus \{\varnothing\},$$
$$\mathcal{P}_{k+1}^*(S) = \mathcal{P}^*\big(\mathcal{P}_k^*(S)\big), \quad k \geq 1,$$

where, for any set X, one writes $\mathcal{P}^*(X) = \mathcal{P}(X) \setminus \{\varnothing\}$.

Example 1.6 (Nonempty Iterated Power Set in Beverage Selection)**.** Let $S = \{\mathsf{Tea},\ \mathsf{Coffee}\}$ represent two drink options. Then:

$$\mathcal{P}_1^*(S) = \mathcal{P}(S) \setminus \{\varnothing\}$$
$$= \big\{\{\mathsf{Tea}\},\ \{\mathsf{Coffee}\},\ \{\mathsf{Tea}, \mathsf{Coffee}\}\big\}.$$

The second-level nonempty iterated power set is

$$\mathcal{P}_2^*(S) = \mathcal{P}^*\left(\mathcal{P}_1^*(S)\right)$$
$$= \Big\{\{\{\mathsf{Tea}\}\},\ \{\{\mathsf{Coffee}\}\},\ \{\{\mathsf{Tea}, \mathsf{Coffee}\}\},\ \{\{\mathsf{Tea}\}, \{\mathsf{Coffee}\}\},$$
$$\{\{\mathsf{Tea}\}, \{\mathsf{Tea}, \mathsf{Coffee}\}\},\ \{\{\mathsf{Coffee}\}, \{\mathsf{Tea}, \mathsf{Coffee}\}\},\ \{\{\mathsf{Tea}\}, \{\mathsf{Coffee}\}, \{\mathsf{Tea}, \mathsf{Coffee}\}\}\Big\}.$$

For instance, $\{\{\mathsf{Tea}\}, \{\mathsf{Coffee}\}\} \in \mathcal{P}_2^*(S)$ can model offering both a “tea-only” service and a “coffee-only” service as distinct package options. One may continue to $\mathcal{P}_3^*(S) = \mathcal{P}^*(\mathcal{P}_2^*(S))$, whose elements are nonempty collections of these service-packages.

Example 1.7 (Crop rotations; a concrete nonempty iterated power set for $|S| = 3$)**.** Let the set of candidate crops be

$$S = \{\text{Wheat},\ \text{Corn},\ \text{Soybean}\}.$$

Then

$$\mathcal{P}_1^*(S) = \big\{\{\text{Wheat}\},\ \{\text{Corn}\},\ \{\text{Soybean}\},\ \{\text{Wheat}, \text{Corn}\},\ \{\text{Wheat}, \text{Soybean}\},$$
$$\{\text{Corn}, \text{Soybean}\},\ \{\text{Wheat}, \text{Corn}, \text{Soybean}\}\big\},$$

so $|\mathcal{P}_1^*(S)| = 2^3 - 1 = 7$. The next level is

$$\mathcal{P}_2^*(S) = \mathcal{P}\big(\mathcal{P}_1^*(S)\big) \setminus \{\varnothing\},$$
$$|\mathcal{P}_2^*(S)| = 2^{\,7} - 1 = 127.$$

Two explicit elements of $\mathcal{P}_2^*(S)$ are

$$A_1 = \big\{\{\text{Wheat}\},\ \{\text{Corn}, \text{Soybean}\}\big\}, \qquad A_2 = \big\{\{\text{Wheat}, \text{Corn}\},\ \{\text{Wheat}, \text{Soybean}\},\ \{\text{Wheat}, \text{Corn}, \text{Soybean}\}\big\}.$$

Agricultural interpretation: elements of $\mathcal{P}_1^*(S)$ are concrete rotation blocks (e.g. $\{\text{Corn}, \text{Soybean}\}$ for a two-year rotation); elements of $\mathcal{P}_2^*(S)$ are *sets of such blocks* (e.g. portfolios of rotations assigned to different fields or planning scenarios compared at the farm scale).

Example 1.8 (Crop rotations; a concrete nonempty iterated power set for $|S| = 3$)**.** Let the set of candidate crops be

$$S = \{\text{Wheat},\ \text{Corn},\ \text{Soybean}\}.$$

Then

$$\mathcal{P}_1^*(S) = \big\{\{\text{Wheat}\},\ \{\text{Corn}\},\ \{\text{Soybean}\},\ \{\text{Wheat}, \text{Corn}\},\ \{\text{Wheat}, \text{Soybean}\},$$
$$\{\text{Corn}, \text{Soybean}\},\ \{\text{Wheat}, \text{Corn}, \text{Soybean}\}\big\},$$

so $|\mathcal{P}_1^*(S)| = 2^3 - 1 = 7$. The next level is

$$\mathcal{P}_2^*(S) = \mathcal{P}\big(\mathcal{P}_1^*(S)\big) \setminus \{\varnothing\}, \qquad |\mathcal{P}_2^*(S)| = 2^{\,7} - 1 = 127.$$

Two explicit elements of $\mathcal{P}_2^*(S)$ are

$$A_1 = \big\{\{\text{Wheat}\},\ \{\text{Corn}, \text{Soybean}\}\big\}, \qquad A_2 = \big\{\{\text{Wheat}, \text{Corn}\},\ \{\text{Wheat}, \text{Soybean}\},\ \{\text{Wheat}, \text{Corn}, \text{Soybean}\}\big\}.$$

Agricultural interpretation: elements of $\mathcal{P}_1^*(S)$ are concrete rotation blocks (e.g. $\{\text{Corn}, \text{Soybean}\}$ for a two-year rotation); elements of $\mathcal{P}_2^*(S)$ are *sets of such blocks* (e.g. portfolios of rotations assigned to different fields or planning scenarios compared at the farm scale).

Example 1.9 (Field operations (management bundles); a concrete nonempty iterated power set for $|S| = 4$)**.** Let

$$S = \{\text{Tillage},\ \text{Sowing},\ \text{Irrigation},\ \text{Fertilization}\}.$$

Then

$$\begin{aligned}\mathcal{P}_1^*(S) = \{&\{\text{Tillage}\},\ \{\text{Sowing}\},\ \{\text{Irrigation}\},\ \{\text{Fertilization}\},\\ &\{\text{Tillage}, \text{Sowing}\},\ \{\text{Tillage}, \text{Irrigation}\},\ \{\text{Tillage}, \text{Fertilization}\},\\ &\{\text{Sowing}, \text{Irrigation}\},\ \{\text{Sowing}, \text{Fertilization}\},\ \{\text{Irrigation}, \text{Fertilization}\},\\ &\{\text{Tillage}, \text{Sowing}, \text{Irrigation}\},\ \{\text{Tillage}, \text{Sowing}, \text{Fertilization}\},\\ &\{\text{Tillage}, \text{Irrigation}, \text{Fertilization}\},\ \{\text{Sowing}, \text{Irrigation}, \text{Fertilization}\},\\ &\{\text{Tillage}, \text{Sowing}, \text{Irrigation}, \text{Fertilization}\}\}.\end{aligned}$$

so $|\mathcal{P}_1^*(S)| = 2^4 - 1 = 15$ and

$$|\mathcal{P}_2^*(S)| = 2^{\,15} - 1 = 32767.$$

Two explicit elements of $\mathcal{P}_2^*(S)$ are

$$B_1 = \big\{\{\text{Tillage}, \text{Sowing}\},\ \{\text{Irrigation}\}\big\},$$
$$B_2 = \big\{\{\text{Sowing}, \text{Fertilization}\},\ \{\text{Irrigation}, \text{Fertilization}\},\ \{\text{Tillage}, \text{Sowing}, \text{Irrigation}, \text{Fertilization}\}\big\}.$$

Agricultural interpretation: members of $\mathcal{P}_1^*(S)$ are operation bundles applied together (e.g. $\{\text{Tillage}, \text{Sowing}\}$ at establishment); members of $\mathcal{P}_2^*(S)$ are *sets of bundles* (e.g. alternative management templates to be compared across fields, seasons, or irrigation availability scenarios).

1.2| Hypergraphs and SuperHypergraphs

Hypergraphs generalize ordinary graphs by allowing each *hyperedge* to join an arbitrary nonempty subset of vertices, thereby modeling higher-order relations among elements [9, 10, 11]. A *SuperHyperGraph* further extends this idea by incorporating iterated powerset structures, enabling multi-layered, self-referential connections among hyperedges [12, 13, 14, 15].

Definition 1.10 (Hypergraph)**.** [9, 16] Let V be a finite set of *vertices*. A *hypergraph* is a pair

$$H = \big(V,\ E\big), \qquad E \subseteq \mathcal{P}(V) \setminus \{\varnothing\},$$

where each element of E is called a *hyperedge*. No restriction is imposed on the size of a hyperedge.

Example 1.11 (Agricultural Hypergraph: crop rotations across fields)**.** Let the vertex set be candidate crops

$$V = \{\text{Wheat},\ \text{Corn},\ \text{Soybean},\ \text{Canola}\}.$$

Each field's multi-year rotation induces a hyperedge equal to the set of crops appearing in that field's plan:

$$\begin{aligned} e_{\text{Field 1}} &= \{\text{Wheat}, \text{Soybean}\}, \\ e_{\text{Field 2}} &= \{\text{Corn}, \text{Soybean}, \text{Wheat}\}, \\ e_{\text{Field 3}} &= \{\text{Canola}\}. \end{aligned}$$

Then $H = (V, E)$ with $E = \{e_{\text{Field 1}}, e_{\text{Field 2}}, e_{\text{Field 3}}\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$ is a hypergraph per Definition 1.10. Each hyperedge captures a (possibly multi-year) rotation used on a specific field.

Definition 1.12 (n-SuperHyperGraph)**.** [12] Let V_0 be a finite *base set*. Define the iterated powersets by

$$\mathcal{P}^0(V_0) = V_0,$$
$$\mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big), \quad k \geq 0.$$

For a fixed $n \geq 1$, an *n-SuperHyperGraph* is a pair

$$\mathrm{SHG}^{(n)} = \big(V,\ E\big),$$

where

$$V,\, E \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad V \neq \varnothing,\ E \neq \varnothing.$$

Elements of V are called *n-supervertices* and elements of E are called *n-superedges*.

Example 1.13 (Corporate Hierarchy as a 2-SuperHyperGraph)**.** Let the set of employees be

$$V_0 = \{A, B, C, D\},$$

with $A = $ Alice, $B = $ Bob, $C = $ Carol, and $D = $ Dave. First-level subsets (teams) are chosen as

$$T_1 = \{A, B\}, \quad T_2 = \{C, D\},$$

so $T_1, T_2 \in \mathcal{P}(V_0)$. Next, define two departments as elements of the second-level iterated power set:

$$D_1 = \{\, T_1\}, \quad D_2 = \{\, T_2\}, \quad \text{so } \{D_1, D_2\} \subseteq \mathcal{P}^2(V_0).$$

Thus we set

$$V_2 = \{\, D_1, D_2\} \subseteq \mathcal{P}^2(V_0), \qquad E_2 = \big\{\{\, D_1, D_2\}\big\} \subseteq \mathcal{P}^2(V_0).$$

The pair

$$F^{(2)} \ = \ \big(V_2,\, E_2\big)$$

is a 2-SuperHyperGraph encoding the hierarchy: employees $\to$ teams $\to$ departments $\to$ the company division.

Example 1.14 (Agricultural n=1 SuperHyperGraph: management bundles and protocols)**.** Let the base set collect crops, inputs, and practices

$$V_0 = \{\text{Wheat}, \text{ Corn}, \text{ Urea (N)}, \text{ DAP (P)}, \text{ Drip}, \text{ Sprinkler}, \text{ HerbicideA}\}.$$

Fix $n = 1$. Choose 1-supervertices (nonempty subsets of V_0) that represent reusable *management bundles*:

$$v_1 = \{\text{Wheat}, \text{Urea (N)}\}, \quad v_2 = \{\text{Corn}, \text{DAP (P)}\}, \quad v_3 = \{\text{Drip}\}, \quad v_4 = \{\text{HerbicideA}, \text{Sprinkler}\}.$$

Set $V = \{v_1, v_2, v_3, v_4\} \subseteq \mathcal{P}^1(V_0)$. Define superedges (each a nonempty subset of V) to encode whole-field protocols:

$$e_1 = \{v_1, v_3\} \quad \text{(Wheat + N with drip irrigation)}, \qquad e_2 = \{v_2, v_4\} \quad \text{(Corn + P with herbicide under sprinkler)}.$$

Then $\text{SHG}^{(1)} = (V, E)$ with $E = \{e_1, e_2\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$ is an n=1 SuperHyperGraph per Definition 1.12. Here, supervertices are management bundles (collections of base actions/resources), and superedges bundle those bundles into implementable protocols.

1.3| Food Web

A food web is a directed graph representing species as nodes and predator–prey interactions as directed edges in an ecosystem[17, 18, 19, 20, 21].

Definition 1.15 (Food Web)**.** [17, 18] Let V be a finite set whose elements represent biological species (or trophic groups) in an ecosystem. A *food web* is the directed graph

$$G = (V, E),$$

where the edge set $E \subseteq V \times V$ satisfies

$$(u, v) \in E \quad \Longleftrightarrow \quad \text{species } u \text{ preys upon species } v.$$

We require additionally that

- There are no self-loops: $(v, v) \notin E$ for all $v \in V$.
- There are no parallel edges: E is a set (not a multiset).

Thus G encodes all direct predator–prey relationships among the species in the ecosystem.

Example 1.16 (Simple Pond Food Web)**.** Consider a small pond ecosystem with five trophic levels:

$$V = \{\, A,\ Z,\ S,\ B,\ D\},$$

where

- A = Algae
- Z = Zooplankton
- S = Small Fish
- B = Big Fish
- D = Duck

The predator–prey relationships are captured by

$$E = \{\, (Z, A),\ (S, Z),\ (B, S),\ (D, B),\ (D, S)\,\},$$

meaning:

$$\begin{aligned}
Z &\to A && \text{(zooplankton eats algae)},\\
S &\to Z && \text{(small fish eat zooplankton)},\\
B &\to S && \text{(big fish eat small fish)},\\
D &\to B && \text{(duck eats big fish)},\\
D &\to S && \text{(duck also eats small fish)}.
\end{aligned}$$

Thus the directed graph $G = (V, E)$ illustrates a simple pond food web.

2| Main Results

In this section, we formally define the concepts of the Food HyperWeb and the Food SuperHyperWeb, and briefly examine their structural properties.

2.1| Food HyperWeb

Food HyperWeb is a hypergraph whose vertices represent species and whose hyperedges correspond to each predator's unique complete prey set.

Definition 2.1 (Base Food Web)**.** Let V_0 be a finite set of species and let

$$G = (V_0, E_0)$$

be the directed graph (the *Food Web*) where $(u, v) \in E_0$ if and only if species u preys upon species v.

Definition 2.2 (Food HyperWeb)**.** Let $G = (V_0, E_0)$ be a Food Web. Define the *Food HyperWeb*

$$H = (V_1, E_1)$$

by

$$V_1 = V_0, \qquad E_1 = \left\{ e_u \subseteq V_0 : e_u = \{ v \in V_0 : (u, v) \in E_0 \},\ e_u \neq \varnothing \right\}.$$

Each hyperedge e_u collects all prey of predator u.

Example 2.3 (Simple Pond Food HyperWeb)**.** Consider the same small pond ecosystem as in the Food Web example:

$$V_0 = \{ A,\ Z,\ S,\ B,\ D\},$$

where

$$E_0 = \{(Z, A),\ (S, Z),\ (B, S),\ (D, B),\ (D, S)\},$$

with species labels:

- A: Algae
- Z: Zooplankton
- S: Small Fish
- B: Big Fish
- D: Duck

The Food HyperWeb $H = (V_1, E_1)$ is obtained by grouping each predator's prey into a hyperedge:

$$V_1 = V_0, \qquad E_1 = \{ e_Z, e_S, e_B, e_D\},$$

where

$$\begin{aligned} e_Z &= \{ A\}, && \text{(zooplankton eats algae);} \\ e_S &= \{ Z\}, && \text{(small fish eat zooplankton);} \\ e_B &= \{ S\}, && \text{(big fish eat small fish);} \\ e_D &= \{ B, S\}, && \text{(duck eats big fish and small fish).} \end{aligned}$$

Thus H is the hypergraph whose vertices are the five species and whose hyperedges capture each predator's full prey set.

Example 2.4 (Marine Food HyperWeb)**.** Consider a simple marine ecosystem with six species:

$$V_0 = \{ P,\ Z,\ K,\ F,\ S,\ H\},$$

where

- P: Phytoplankton
- Z: Zooplankton
- K: Krill
- F: Small Fish
- S: Seal
- H: Shark

The predator–prey relations (the Food Web $G = (V_0, E_0)$) are

$$E_0 = \{ (Z, P),\ (K, Z),\ (F, K),\ (F, Z),\ (S, F),\ (S, K),\ (H, S),\ (H, F) \}.$$

From this we form the Food HyperWeb $H = (V_1, E_1)$ with $V_1 = V_0$ and

$$E_1 = \{ e_Z, e_K, e_F, e_S, e_H \},$$

where each hyperedge collects all prey of a given predator:

$$\begin{aligned}
e_Z &= \{ P \}, && \text{(zooplankton eats phytoplankton);}\\
e_K &= \{ Z \}, && \text{(krill eats zooplankton);}\\
e_F &= \{ K,\ Z \}, && \text{(small fish eat krill and zooplankton);}\\
e_S &= \{ F,\ K \}, && \text{(seal eats small fish and krill);}\\
e_H &= \{ S,\ F \}, && \text{(shark eats seal and small fish).}
\end{aligned}$$

Thus H is the hypergraph whose vertices are the six marine species and whose hyperedges exactly describe each predator's full prey set.

Theorem 2.5 (Food HyperWeb is a Hypergraph)**.** *Let $H = (V_1, E_1)$ be the Food HyperWeb constructed from a Food Web $G = (V_0, E_0)$ as in Definition 2.1. Then*

$$E_1 \subseteq \mathcal{P}(V_1) \setminus \{\varnothing\}, \qquad V_1 \text{ is finite,}$$

so H satisfies the axioms of a finite hypergraph.

Proof: By definition $V_1 = V_0$ is finite. Each hyperedge

$$e_u = \{ v \in V_1 : (u, v) \in E_0 \}$$

is nonempty exactly when u preys on at least one species, hence $e_u \in \mathcal{P}(V_1) \setminus \{\varnothing\}$. Moreover, the assignment $u \mapsto e_u$ is injective, so there are no duplicate hyperedges. Thus H meets the standard definition of a finite hypergraph [10]. □

Theorem 2.6 (Reconstruction of Food Web by Flattening)**.** *The original Food Web $G = (V_0, E_0)$ is recovered from the Food HyperWeb $H = (V_1, E_1)$ via*

$$E_0 = \Big\{ (u, v) \in V_1 \times V_1 : v \in e_u,\ e_u \in E_1 \Big\}.$$

Proof: By construction $e_u = \{v : (u, v) \in E_0\}$. Hence

$$(u, v) \in E_0 \iff v \in e_u,$$

so the directed edges of G coincide exactly with the incidences of vertices in hyperedges of H. Therefore flattening H recovers E_0. □

2.2| Food SuperHyperWeb

Food SuperHyperWeb is an n-superhypergraph whose vertices are iterated species subsets and whose hyperedges encode complete multilevel hierarchical predator–prey relationships.

Definition 2.7 (Food n-SuperHyperWeb)**.** Let $G = (V_0, E_0)$ be a finite directed graph (the *Food Web*), and for each $u \in V_0$ write

$$P(u) \;=\; \{\, v \in V_0 : (u, v) \in E_0 \}$$

for its *prey set*. Fix an integer $n \geq 1$, and let $\mathcal{P}^n(V_0)$ denote the n-fold iterated powerset of V_0. Define

$$V_n \;=\; \mathcal{P}^n(V_0), \qquad E_n \;=\; \big\{\, e_u^{(n)} : u \in V_0,\ P(u) \neq \varnothing,\ e_u^{(n)} = \mathcal{P}^n\big(P(u)\big) \big\}.$$

Then the pair

$$F^{(n)} = \big(V_n,\ E_n\big)$$

is called the *Food n-SuperHyperWeb*.

Example 2.8 (Forest Ecosystem as a Food 2-SuperHyperWeb)**.** Let the species set be

$$V_0 = \{G,\, M,\, O,\, F\},$$

where

$$G = \text{Grass}, \quad M = \text{Mouse}, \quad O = \text{Owl}, \quad F = \text{Fox}.$$

The Food Web $G = (V_0, E_0)$ has predator–prey arcs

$$E_0 = \{(M, G),\, (O, M),\, (F, O),\, (F, M)\}.$$

For each $u \in V_0$, its prey set is

$$P(M) = \{G\},\ P(O) = \{M\},\ P(F) = \{O,\, M\},\ P(G) = \varnothing.$$

Fix $n = 2$. Then

$$V_2 = \mathcal{P}^2(V_0), \qquad E_2 = \{\, e_u^{(2)} : u \in \{M, O, F\}\},$$

with

$$e_M^{(2)} = \mathcal{P}^2\big(P(M)\big) = \mathcal{P}^2(\{G\}) = \big\{\emptyset, \{\emptyset\}, \{\{G\}\}, \{\emptyset, \{G\}\}\big\},$$
$$e_O^{(2)} = \mathcal{P}^2(\{M\}) = \big\{\emptyset, \{\emptyset\}, \{\{M\}\}, \{\emptyset, \{M\}\}\big\},$$
$$e_F^{(2)} = \mathcal{P}^2\big(\{O, M\}\big),$$

which has $2^{2^2} = 16$ elements (all subsets of $\mathcal{P}(\{O, M\})$). Thus

$$F^{(2)} = \big(V_2,\ E_2\big)$$

is the Food 2-SuperHyperWeb encoding the multi-level trophic structure:

$$\text{species} \;\to\; \text{prey sets} \;\xrightarrow{\mathcal{P}}\; \text{collections of prey-sets}.$$

Example 2.9 (Agricultural Food 2-SuperHyperWeb)**.** Let the set of trophic species be

$$V_0 = \{G,\, I,\, C,\, Co,\, H\},$$

where

$$G = \text{Grass}, \quad I = \text{Insect}, \quad C = \text{Chicken}, \quad Co = \text{Cattle}, \quad H = \text{Human}.$$

The Food Web $G = (V_0, E_0)$ has predator–prey arcs

$$E_0 = \{(I, G),\, (C, I),\, (Co, G),\, (H, C),\, (H, Co)\}.$$

Hence each prey set is

$$P(I) = \{G\},\ P(C) = \{I\},\ P(Co) = \{G\},\ P(H) = \{C, Co\}.$$

Fix $n = 2$. Then

$$V_2 = \mathcal{P}^2(V_0), \qquad E_2 = \{\, e_u^{(2)} : u \in \{I, C, Co, H\}\},$$

with

$$e_I^{(2)} = \mathcal{P}^2\big(\{G\}\big) = \big\{\varnothing, \{\varnothing\}, \{\{G\}\}, \{\varnothing, \{G\}\}\big\},$$

$$e_C^{(2)} = \mathcal{P}^2\big(\{I\}\big) = \big\{\varnothing, \{\varnothing\}, \{\{I\}\}, \{\varnothing, \{I\}\}\big\},$$
$$e_{Co}^{(2)} = \mathcal{P}^2\big(\{G\}\big) \quad \text{(same as } e_I^{(2)}\text{)},$$
$$e_H^{(2)} = \mathcal{P}^2\big(\{C, Co\}\big),$$

which has $2^{2^2} = 16$ elements, each representing a possible "meal bundle" drawn from the basic prey sets $\{C, Co\}$. Therefore

$$F^{(2)} = \big(V_2,\ E_2\big)$$

is the Food 2-SuperHyperWeb encoding both primary producer consumption and human dietary options in a two-layer superhypergraph structure.

Theorem 2.10. *$F^{(n)} = (V_n, E_n)$ is an n-SuperHyperGraph.*

Proof: By construction V_n is a finite set and each hyperedge

$$e_u^{(n)} = \mathcal{P}^n\big(P(u)\big)$$

is a nonempty element of $\mathcal{P}^n(V_0)$. Distinct predators u yield distinct hyperedges, so there are no duplicates. Hence $F^{(n)}$ satisfies the definition of an n-SuperHyperGraph. □

Theorem 2.11 (Generalization of Food HyperWeb and Food Web)**.** *Define the "full flattening" of a nested set by taking the union of all its elements at every level. Since*

$$e_u^{(n)} \ = \ \mathcal{P}^n\big(P(u)\big),$$

flattening $e_u^{(n)}$ recovers exactly the prey set $P(u)$. Therefore

$$\big\{\, P(u) : u \in V_0,\ P(u) \neq \varnothing \big\} = E_1 \quad \textit{and} \quad \big\{(u, v) : v \in P(u)\big\} = E_0,$$

showing that $F^{(n)}$ simultaneously generalizes the Food HyperWeb $H = (V_0, E_1)$ and the original Food Web $G = (V_0, E_0)$.

Proof: Immediate from the identity $\bigcup_{X \in e_u^{(n)}} \bigcup \cdots \bigcup X = P(u)$ and the definitions of E_1 and E_0. □

3| Conclusion and Future Works

In this paper, we introduced two novel extensions of classical food webs: the *Food HyperWeb*, which encodes each predator's entire prey set as a hyperedge, and the *Food n-SuperHyperWeb*, which embeds multilevel trophic relationships within an n-fold superhypergraph structure.

For future work, we plan to explore practical applications of these models in real ecological and agricultural networks. Additionally, we aim to extend the framework using uncertainty-based approaches such as Fuzzy Sets [22, 23], Intuitionistic Fuzzy Sets [24, 25], HyperFuzzy Sets[26, 27], Bipolar Fuzzy Sets[28, 29], Neutrosophic Sets[30, 31, 32], Hesitant Fuzzy Sets[33, 34], and Plithogenic Sets[35, 36] to better handle ambiguity and incomplete information in ecological systems.

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Data Availability

This manuscript presents purely conceptual work without empirical data. Scholars interested in these ideas are invited to undertake experimental or case-study research to substantiate and extend the proposed frameworks.

Ethical Approval

This paper involves no human or animal subjects and thus did not require ethics committee review or approval.

Conflicts of Interest

The authors declare that there are no competing interests concerning the content or publication of this article.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Disclaimer

The theoretical models and propositions herein have not yet been subjected to practical validation. Readers should independently verify all citations and be aware that inadvertent inaccuracies may remain. The opinions expressed are those of the authors and do not necessarily represent the views of affiliated organizations.

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